Chapter 5

Differential Amplifiers

5.1 Basics

Differential amplifiers (DA) are important building block of many linear/analog IC units - especially operational amplifiers (Op amp). A basic DA circuit is given in Fig. 5.1.

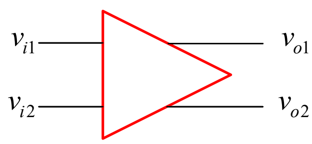


Figure 5.1: Differential amplifier symbol.

DA has two modes of operation, with some variations as shown in Table 5.1. In common mode (CM), DA inputs are connected to the same signal, while in differential mode (DM) two signals (basically) are separately connected to the two input terminals. DAs are characterized by having a large differential mode gain compared to a very small gain in common mode.

Differential mode (DM): The input to the DA is the difference between its two input signals ($\mathrm{v_{id}} = \mathrm{v_{i1}} - \mathrm{v_{i2}}$) while the output can be taken in the following two ways:

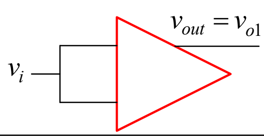
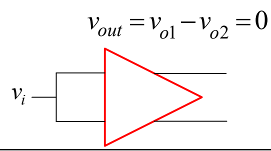
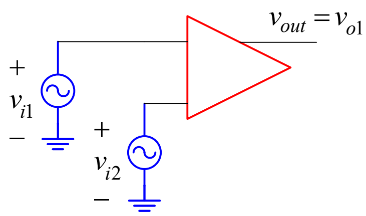
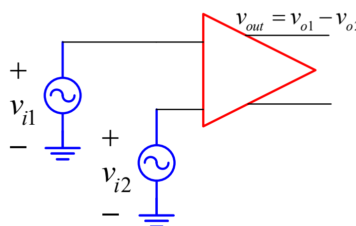
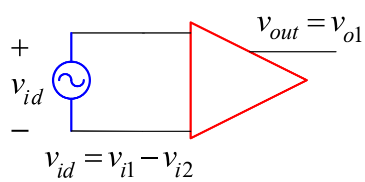
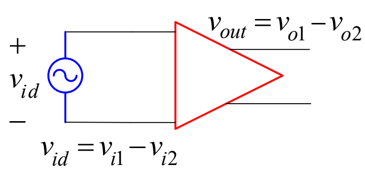
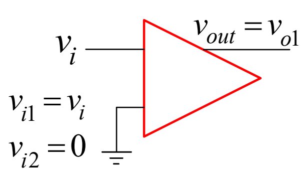
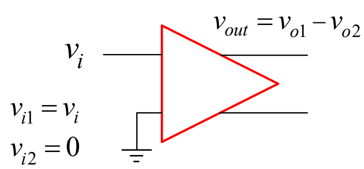
- Difference between two outputs ($v_{od} = v_{o1} - v_{o2}$) or ($v_{od} = v_{o2} - v_{o1}$), then this is called double ended configuration and the gain is A_{dm}.
- Between one output and ground ($v_{od} = v_{o1}$) or ($v_{od} = v_{o2}$) then it is single configuration ended with a gain (A_v)

Note that one of the inputs may be set to zero (grounded) but it is still a DM amplifier.

Common mode (CM): The two input terminals are tied up together ($v_i = v_{i1} = v_{i2}$) while the output can be taken in the following two ways:

- Between two terminals, then ($v_0 = 0$), since both outputs have value the same.
- From a single terminal (single ended), voltage gain is referred to as A_{cm}.

Table 5.1: Differential Amplifiers – Mode of Operation.

Mode of Operation	**Single Ended Configuration**	**Double Ended Configuration**	**Comments**
Common mode (CM)			Double ended is not used, as output voltage is zero!
			This is the basic configuration.
Differential Mode (DM)			Normally used for Analysis.
			Input 2 is zero, but still, it is a differential mode!

5.2 DC Analysis of Differential amplifier

A BJT differential amplifier circuit is shown in Fig. 5.2. The two transistors are matched, i.e., they have the same β. Transistors operate in the active region and are not allowed to enter saturation. DC analysis is needed to calculate transistor's small signal parameters that can be utilized in calculating DA ac qualities such as voltage gain. DC analysis is simplified with the use of constant current source biasing. This will be illustrated with the following example. In DC analysis, both inputs of the DA are connected to the ground (input signal is set to zero).

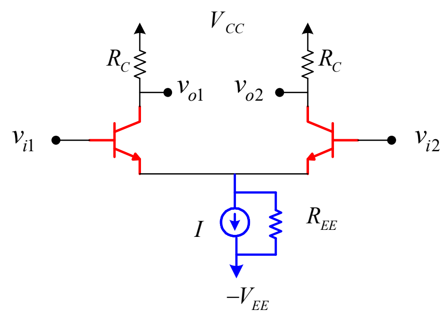


Figure 5.2: BJT differential amplifier.

Example 5.1 DA DC analysis

For the DA circuit shown in Fig. 5.3, find g_m, r_e, r_π if $\beta = 100$.

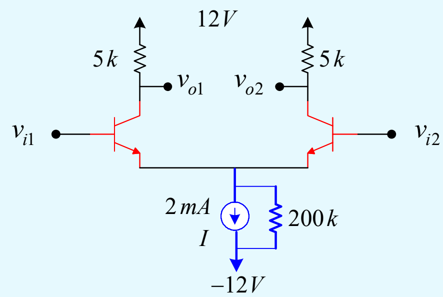


Figure 5.3: DC analysis for Example 5.1.

Solution

In DC analysis, both inputs of the DA are connected to ground (input signal is zero).

$$I_{E1} = I_{E2} = \frac{I}{2} = \frac{2}{2} = 1 \text{ mA}$$

$$I_{C1} = I_{C2} = I_C \approx I_{E1} = \frac{I}{2} = 1 \text{ mA}$$

$$g_m = \frac{I_c}{V_T} = \frac{1}{25} = 40 \text{ mS}$$

$$r_e = \frac{V_T}{I_{E1}} = \frac{25}{1} = 25\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.50 \text{ k}\Omega$$

5.3 AC Analysis of Differential amplifier

From dc conditions, g_m and r_π or r_e are calculated. The general case of a DA with two sources is shown in Fig. 5.4.

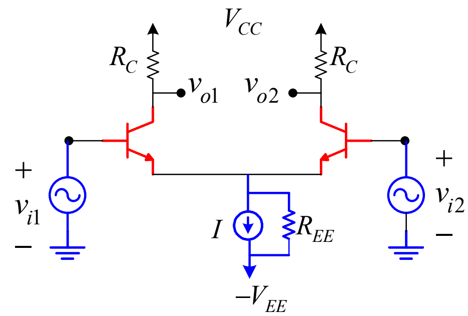


Figure 5.4: General DA Circuit.

Since the dc current source is open at ac, then we are left with its output resistance only ($\mathrm{R_{EE}}$). The transistors are replaced by their equivalent circuits, Fig. 5.5, from which the gain is calculated. Note that currents shown are ac quantities which are variations on DC currents! The current source in the transistor equivalent circuit can be either ($g_\mathrm{m}\mathrm{v_{be}}$) or ($\beta \mathrm{i_b}$), but both give the same answer.

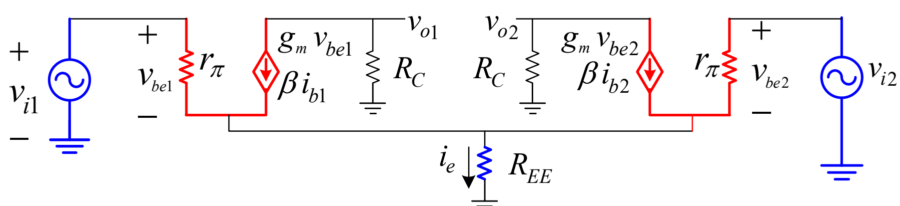


Figure 5.5: DA general equivalent circuit.

5.3.1 Differential Mode - Double ended

The basic configuration is given in Fig. 5.4 and its equivalent circuit is given in Fig. 5.5. This DA is actually connected as shown in Fig. 5.6, where the input signal is the difference between the two inputs, i.e., $v_{id} = v_{i1} - v_{i2}$. Its equivalent circuit is shown in Fig. 5.7.

Since the terminals v_{i1} and v_{i2} form one complete circuit, then currents flowing through each one of them should be the same, i.e.:

$$i_{b1} = i_{b2} = i_b \tag{5.1}$$

Since the base currents have the same value and both transistors have the same β, then their collector currents have the same value (βi_b), therefore their emitter currents must have the same value [$(\beta+1)i_b$]. This means that the current flowing in emitter resistance R_{EE} equals zero. therefore we can remove it from the equivalent circuit as it has no effect on ac performance of the circuit'. Note that i_{be2} is shown going out of Q_2 base. This may look illogical, but it is not. These currents

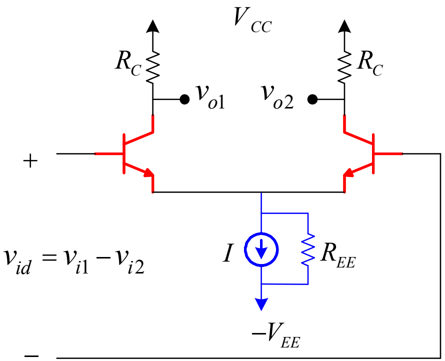


Figure 5.6: Double ended DA circuit.

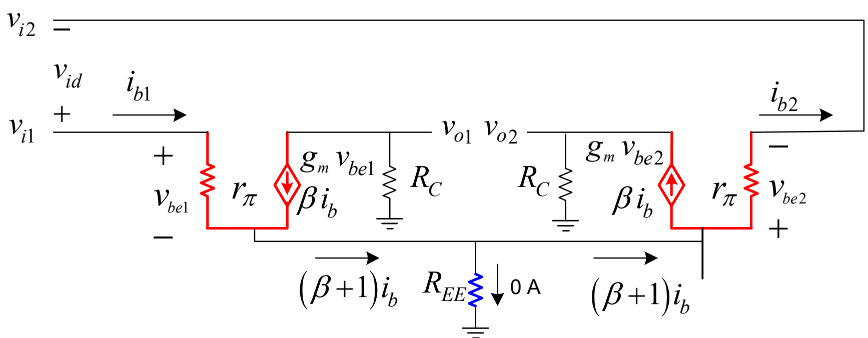


Figure 5.7: Double ended DA – equivalent circuit.

shown in the circuit are ac currents, which are superimposed on the dc currents. The ac currents represent only small variations on the overall currents which do not change their directions.

Since transistors are matched then their base-emitter resistances are the same, and hence:

$$v_{be1} = i_b r_\pi = +v_{be} \qquad v_{be2} = -i_b r_\pi = -v_{be} \tag{5.2}$$

Therefore:

$$v_{id} = v_{i1} - v_{i2} = v_{be1} + v_{eb2} = v_{be1} - v_{be2} = v_{be} - (-v_{be}) = v_{be} + v_{be} = 2\ v_{be} \tag{5.3}$$

$$v_{o1} = -g_m v_{be} R_C$$

$$v_{o2} = +g_m v_{be} R_C \quad \text{(Note the direction of collector currents)}$$

$$A_{dm} = \frac{v_{o1} - v_{o2}}{v_{i1} - v_{i2}} = \frac{v_{od}}{v_{id}} = \frac{-g_m v_{be} R_C - (g_m v_{be} R_C)}{2v_{be}} = -\frac{2g_m v_{be} R_C}{2v_{be}}$$

$$A_{dm} = -g_m R_C \tag{5.4}$$

But $g_m = \dfrac{\beta}{r_\pi}$, therefore, this result can be presented in another format:

$$A_{dm} = -\frac{\beta}{r_\pi} R_C \tag{5.5}$$

For high β:

$$\mathrm{r}_\pi = (\beta + 1) r_e \approx \beta r_e$$
$$\therefore A_{dm} = \frac{v_{od}}{v_{id}} \approx -\frac{\beta R_c}{\beta r_e} = -\frac{R_c}{r_e} \tag{5.6}$$

The differential input resistance is:

$$R_{id} = \frac{v_{id}}{i_{id}} = \frac{2 v_{be}}{i_b} = 2 r_\pi \tag{5.7}$$

To calculate the output differential resistance, the input voltage is set to zero ($v_{id} = 0$), which makes the dependent current sources open-circuited and hence resulting the circuit shown in Fig. 5.8. Therefore the resistance seen from the output terminals is simply two collectors' resistors connected in series, i.e.:

$$R_o = 2 R_C \tag{5.8}$$

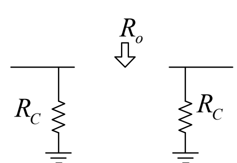


Figure 5.8: Finding output resistance of differential amplifier in common mode.

5.3.2 Differential Mode - Single ended voltage gain

For the circuit given in Fig. 5.7, the output can be taken from either of the two output terminals. Taking the output from terminal 2, (v_{02}), then:

$$A_v = \frac{v_{o2}}{v_{id}} = \frac{g_m v_{be} R_C}{2 v_{be}} = \frac{1}{2} g_m R_C \tag{5.9}$$

But $g_m = \frac{\beta}{r_\pi}$, therefore, this result can be presented in another format:

$$A_{dm} = \frac{1}{2} \beta \frac{R_C}{r_\pi} \tag{5.10}$$

Taking the output from terminal 1, (v_{o1}), then:

$$A_v = \frac{v_{o1}}{v_{id}} = -\frac{1}{2} g_m R_C \tag{5.11}$$

$$A_v = \frac{v_{o1}}{v_{id}} = -\frac{1}{2} \beta \frac{R_C}{r_\pi} \tag{5.12}$$

If one of the inputs is connected to the ground, Fig. 5.9, then we still have a DM single ended DA. Its ac equivalent circuit is shown in Fig. 5.10.

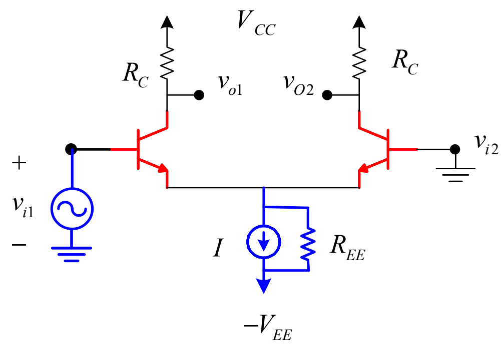


Figure 5.9: DM - Single ended DA.

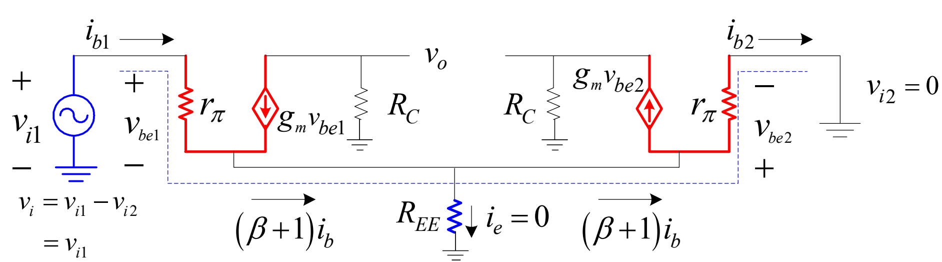


Figure 5.10: DM - Single ended DA equivalent circuit.

The output is taken between one terminal and ground. Therefore:

$$v_o = v_{o1}$$

Applying KVL to the loop shown:

$$v_{i1} = i_{b1} r_\pi + i_{b2} r_\pi$$

But:

$$i_{b1} = i_{b2} = i_b$$

Therefore:

$$v_i = 2 i_b r_\pi = 2 v_{be}$$

$$v_o = -g_m v_{be} R_C$$
$$A_v = \frac{v_o}{v_{id}} = \frac{v_o}{v_{i1}} = -\frac{g_m R_C v_{be}}{2v_{be}} = -\frac{1}{2} g_m R_C \tag{5.13}$$

But:

$$g_m = \frac{\beta}{r_\pi}$$
$$\therefore A_v = \frac{v_o}{v_{id}} = -\frac{1}{2}\frac{\beta}{r_\pi} R_c = -\frac{1}{2}\frac{\beta R_c}{r_\pi} \tag{5.14}$$

Note that the same results could have been obtained using Fig. 5.7 but taking the output from one terminal only. This output is half of the double ended differential output which will give half of the gain as the input is kept the same.

The above result can be approximated as follows:

$$\mathrm{r}_\pi = (\beta + 1) r_e \approx \beta r_e \qquad \text{for } \beta \gg 1$$
$$\therefore A_v = \frac{v_o}{v_i} \approx -\frac{1}{2}\frac{\beta R_c}{\beta r_e} \approx -\frac{R_c}{2 r_e} \tag{5.15}$$

From Fig. 5.10, the input differential resistance is:

$$R_i = \frac{v_i}{i_i} = \frac{2 v_{be}}{i_b} = 2 r_\pi \tag{5.16}$$

The output resistance seen at the output (between output terminal and ground) will be simply the two collector resistor, i.e.:

$$R_o = R_C \tag{5.17}$$

Example 5.2 Differential mode amplifier

Calculate v_0 and single ended voltage gain A_v for the circuit of Example 5.1 if a 2 mV signal is applied between the two input terminals of the DA.

Solution

Using the results obtained earlier in Example 5.1, our dc analysis gave:

$$r_\pi = 2.5 \text{ k}\Omega, \quad g_m = 40 \text{ mS}$$

The ac equivalent circuit is similar to that shown in Fig. 5.10, therefore:

$$A_v = \frac{v_o}{v_{id}} = -\frac{g_m R_C}{2} = -\frac{1}{2} 40 \text{ mS} \times 5 \text{ k}\Omega = -200 \ \text{ V/V}$$

$$v_0 = A_v v_i = 200 \times 2 \text{ (mV)} = 0.4 \text{ V}$$

5.3.3 Common mode DA

This DA is connected as shown in Fig. 5.11, where its two inputs are tied together and connected to the input signal, i.e. $v_{i1} = v_{i2} = v_i$. The DA equivalent circuit is shown in Fig. 5.12. The output is taken from one terminal because if it is taken differentially then $v_o = 0$ since both outputs are equal ($v_{o1} = v_{o2}$).

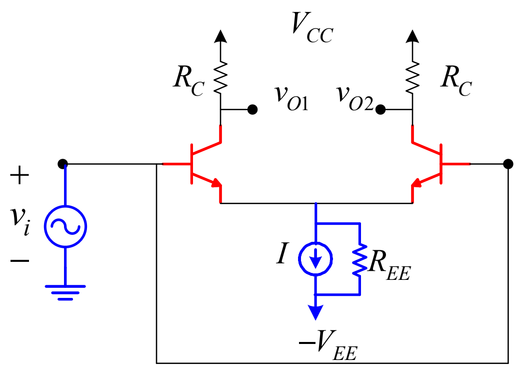


Figure 5.11: Common mode DA configuration.

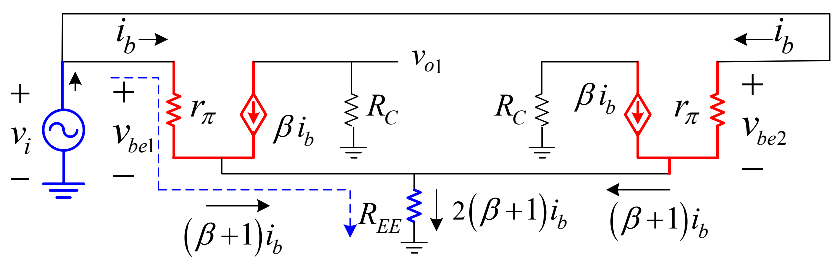


Figure 5.12: Common mode DA equivalent circuit.

$$\begin{aligned} v_i &= i_b r_\pi + V_{R_{EE}} \\ &= i_b r_\pi + 2(\beta+1) i_b R_{EE} = i_b \left[r_\pi + 2(\beta+1) R_{EE} \right] \\ v_o &= -\beta i_b R_c \end{aligned}$$

Therefore:

$$A_{cm} = \frac{v_o}{v_i} = \frac{-\beta i_b R_c}{i_b \left[r_\pi + 2(\beta+1) R_{EE} \right]} = -\frac{\beta R_c}{r_\pi + 2(\beta+1) R_{EE}} \tag{5.18}$$

But:

$$r_\pi \ll 2(\beta+1) R_{EE}$$

$$\therefore A_{cm} \approx \frac{-\beta R_c}{2(\beta+1)R_{EE}} \approx -\frac{\beta R_c}{2\beta R_{EE}} \approx -\frac{R_c}{2R_{EE}} \tag{5.19}$$

A DA stage must have a very small A_{CM}, a term used to measure this is the common mode rejection ratio (CMRR), which is defined as:

$$\begin{aligned} CMRR &= \left|\frac{A_{dm}}{A_{cm}}\right| \qquad V/V \\ &= 20\log\left|\frac{A_{dm}}{A_{cm}}\right| \qquad \text{dB} \end{aligned} \tag{5.20}$$

Note: dB stands for Deci Bell, which will be discussed in more detail later in chapter 8.

Referring to Fig. 5.12, the input resistance is calculated as follows:

$$i_i = 2i_b$$

$$v_i = i_b r_\pi + 2(\beta+1)i_b R_{EE} = i_b\left[r_\pi + 2(\beta+1)R_{EE}\right]$$

$$R_i = \frac{v_i}{i_i} = \frac{i_b\left[r_\pi + 2(\beta+1)R_{EE}\right]}{2i_b}$$

$$R_i = \frac{r_\pi}{2} + (\beta+1)R_{EE} \tag{5.21}$$

But for large β : $\quad \frac{r_\pi}{2} \ll (\beta+1)R_{EE} \quad$ and $\quad \beta+1 \approx \beta \quad$ therefore:

$$R_i \approx \beta R_{EE}$$

The output resistance seen at the output (between output terminal and ground) will be simply the collector resistor, i.e.:

$$R_o = R_C \tag{5.22}$$

Example 5.3 AC analysis of BJT DA

For the differential amplifier shown in Figure 5.13, find:

(a) Differential gain

(b) Common mode gain

(c) CMRR

Solution

This is the same circuit studied earlier, with same components used in Example 5.1, which allows us to apply results obtained earlier directly.

$$r_\pi = 2.5 \text{ k}\Omega, \quad g_m = 40 \text{ mS}$$

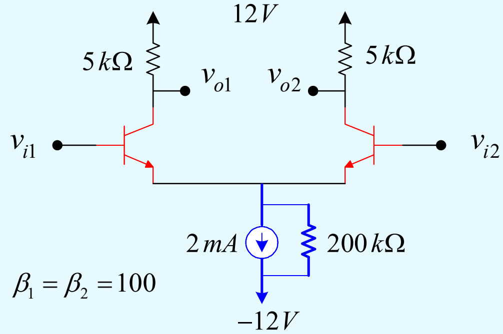


Figure 5.13: AC analysis of BJT DA.

(a) Differential gain:

$$A_{dm} = -g_m R_C = -40 \times 5 = -200$$

(b) Common node gain

Using exact formula:

$$A_{cm} = \frac{v_o}{v_i} = \frac{-\beta R_c}{r_\pi + 2(\beta+1)R_{EE}} = \frac{-100(5)}{2.53 + 2(100+1)200} = 0.012$$

Using approximate formula:

$$A_{cm} = \frac{v_o}{v_i} \approx \frac{R_c}{2R_{EE}} = \frac{5}{2(200)} = 0.0125$$

$$\%\ \text{error} = \frac{0.0125 - 0.012}{0.012} \times 100 = 1\%$$

This is very close to the exact value calculated above.

(c) The common mode rejection ratio:

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{200}{0.0125} = 1.6 \times 10^4$$
$$= 20 \log\left(1.6 \times 10^4\right) = 84.1 \text{ dB}$$

Example 5.4 Differential amplifier driven by Wilson current source.

A differential amplifier (Q_1, Q_2) driven by a Wilson current source (Q_3, Q_4, Q_5), is shown in Fig. 5.14. Calculate:

(a) The input resistance - differential mode

(b) The input resistance - common mode

(c) The output resistance - differential mode

(d) The output resistance - common mode

(e) Differential mode gain

(f) Common mode gin

(g) CMMR

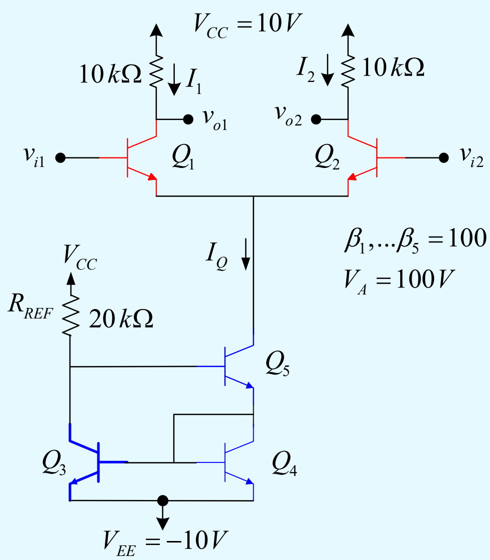


Figure 5.14: Differential amplifier driven by Wilson current source example.

Solution

We need first to find transistors' parameters. For Wilson current source (Q_3, Q_4, Q_5), we have:

$$I_{REF} = \frac{(V_{CC} - V_{EE}) - 2V_{BE}}{R_{REF}} = \frac{[10 - (-10)] - (2 \times 0.7)}{20} = 0.93 \text{ mA}$$

Applying Eq. 4.48 for Wilson current source:

$$I_Q = \frac{I_{REF}}{1 + \frac{2}{\beta^2}} = \frac{0.93}{1 + \frac{2}{100^2}} = 0.93 \text{ mA}$$

$$I_{C1} = I_{C2} = \frac{I_Q}{2} = \frac{0.93}{2} = 0.465 \text{ mA}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.465}{25} = 18.6 \text{ mS}$$

$$g_{m5} = \frac{I_{C5}}{V_T} = \frac{I_Q}{V_T} = \frac{0.93}{25} = 37.2 \text{ mS}$$
$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{18.6} = 5.38 \text{ k}\Omega$$
$$r_{\pi 5} = \frac{\beta_5}{g_{m5}} = \frac{100}{37.2} = 2.69 \text{ k}\Omega$$
$$r_{o5} = \frac{V_A}{I_{C5}} = \frac{100}{0.93} = 107.53 \text{ k}\Omega$$

Applying Eq. 4.56, we get:

$$R_{EE} = R_{o5} = \frac{1}{2}\beta_5 r_{o5} = \frac{1}{2} \times 100 \times 107.53 = 5.377 \text{ M}\Omega$$

(a) The input resistance - differential mode

$$R_{id} = 2r_\pi = 2 \times 5.38 = 10.76 \text{ k}\Omega$$

(b) The input resistance - common mode

$$R_{ic} = r_{\pi 1} + 2(\beta + 1)R_{EE} = 5.38 + 2(100 + 1) \times 5.377 \times 10^3 = 1086 \text{ M}\Omega$$

(c) The output resistance - differential mode

$$R_{od} = 2R_C = 2 \times 10 = 20 \text{ k}\Omega$$

(d) The output resistance - common mode

$$R_{oc} = R_C = 10 \text{ k}\Omega$$

(e) Differential mode gain

$$A_{DM} = -g_m R_C = -18.6 \times 10 = -186$$

(f) Common mode gain

$$A_{CM} = -\frac{\beta R_c}{r_{\pi 1} + 2(\beta + 1)R_{EE}} = -\frac{100 \times 10}{5.38 + 2(100 + 1) \times 5.377 \times 10^3} = 9.21 \times 10^{-4}$$

(g) CMMR

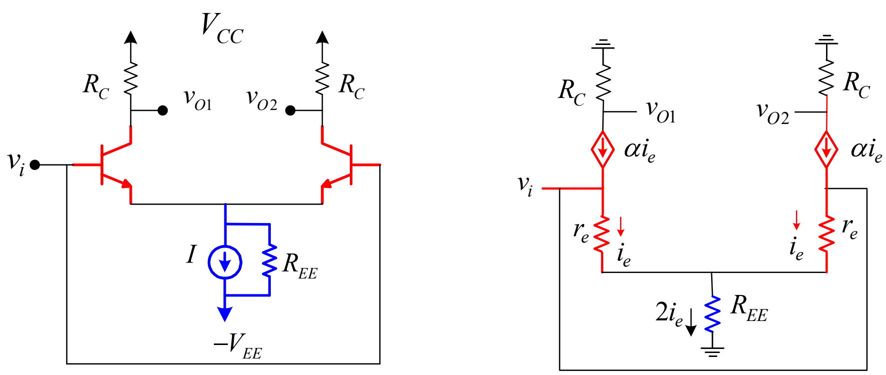
$$CMRR = \left|\frac{A_{DM}}{A_{CM}}\right| = \frac{186}{9.21 \times 10^{-4}} = 2.02 \times 10^5$$

Using approximate formula:

$$R_{ic} \approx 2\beta R_{EE} = 2 \times 5.377 = 1075.4 \ \ \text{M}\Omega$$
$$A_{CM} \approx -\frac{R_c}{2R_{EE}} = -\frac{10}{2 \times 5.377 \times 10^3} = 9.3 \times 10^{-4}$$
$$CMRR = \left|\frac{A_{DM}}{A_{CM}}\right| = \frac{186}{9.3 \times 10^{-4}} = 2.02 \times 10^5$$

5.3.4 Use of T-Model

Using T-model gives the same results obtained earlier with the π-model for the differential and common modes gains. Fig. 5.15 shows the common mode DA together with its equivalent T-model.



(a) Circuit (b) Equivalent circuit

Figure 5.15: Common mode BJT DA and its T model equivalent circuit.

$$v_i = i_e r_e + 2i_e R_{EE} = i_e\left(r_e + 2R_{EE}\right)$$
$$v_{o1} = v_{o2} = -\alpha i_e R_C$$
$$A_{CM} = \frac{v_{o1}}{v_i} = \frac{-\alpha i_e R_C}{i_e\left(r_e + 2R_{EE}\right)} = -\frac{\alpha R_C}{r_e + 2R_{EE}} \tag{5.23}$$

But: $\alpha = \dfrac{\beta}{\beta + 1}$

$$A_{CM} = \frac{v_{o1}}{v_i} = -\frac{\beta R_C}{(\beta+1)\left(r_e + 2R_{EE}\right)} = -\frac{\beta R_C}{(\beta+1)r_e + 2(\beta+1)R_{EE}}$$
$$A_{CM} = \frac{v_{o1}}{v_i} = -\frac{\beta R_C}{r_\pi + 2(\beta+1)R_{EE}} \tag{5.24}$$

Which is the same result obtained using π-model, Eq. 5.18.

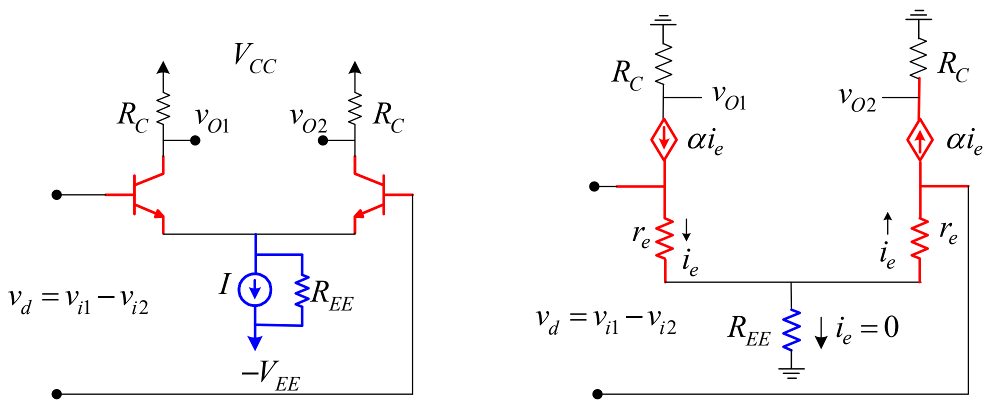
Fig. 5.16 shows the differential mode BJT DA together with its equivalent T-model.

$$v_{id} = v_{i1} - v_{i2} = i_e r_e + i_e r_e = 2i_e r_e$$
$$v_{o1} = -\alpha i_e R_C$$
$$v_{o2} = \alpha i_e R_C$$
$$v_{od} = v_{o1} - v_{o2} = -\alpha i_e R_C - \alpha i_e R_C = -2\alpha i_e R_C$$
$$A_{dm} = \frac{v_{od}}{v_{id}} = \frac{-2\alpha i_e R_C}{2i_e r_e} = -\frac{\alpha}{r_e} R_C = -g_m R_C \tag{5.25}$$

But: $\dfrac{\alpha}{r_e} = g_m$, therefore:

$$A_{dm} = -\beta \frac{R_C}{r_\pi} \tag{5.26}$$

Which are the same results obtained using π-model, Eq. 5.4 and Eq. 5.5.



(a) Circuit (b) Equavilent Circuit

Figure 5.16: Differential mode BJT DA and its equivalent circuit.

Example 5.5 BJT Differential amplifier design

It is required to design a differential amplifier with a differential mode voltage gain of -50. The dc power dissipation is 10 mW. The available power supply is ± 5 V. Available transistors have $\beta = 100$. Assuming $V_T = 25$ mV, find:

(a) The current value of the current source (assume ideal case)

(b) The value of the collector resistance

(c) The input differential resistance

(d) The voltage at each of the output terminals if the input voltage is 10 mV

Solution

Referring to the basic differential amplifier circuit in Fig. 5.16(a), but with an ideal current source, i.e., $R_{EE} = \infty$.

(a) Current value of the current resource

$$P_{dc} = I_{dc}V_{dc} = I_{dc} \times [(+V_{CC}) - (V_{EE})] = I_{dc}\,[(+V_{CC}) - (-V_{CC})] = 2 \times I_{dc} \times V_{CC}$$

$$I_{dc} = \frac{P_{dc}}{2 \times V_{CC}} = \frac{10}{2 \times 5} = 1 \text{ mA}$$

(b) The value of the collector resistance

$$I_C = \frac{I_{dc}}{2} = \frac{1}{2} = 0.5 \text{ mA}$$
$$g_m = \frac{I_C}{V_T} = \frac{0.5}{25} = 20 \text{ mS}$$

Applying

$$R_C = \frac{-A_{DM}}{g_m} = \frac{+50}{20} = 2.5 \text{ k}\Omega$$

(c) Input differential resistance

$$R_o = 2R_C = 2 \times 2.5 = 5 \text{ k}\Omega$$

(d) The voltage at each of the output terminals

$$v_{od} = |A_{DM}| \times v_{id} = 50 \times 10 = 500 \text{ mV}$$
$$v_{C1} = -\frac{1}{2}v_{od} = -\frac{500}{2} = -250 \text{ mV}$$
$$v_{C2} = +\frac{1}{2}v_{od} = \frac{500}{2} = 250 \text{ mV}$$

5.3.5 MOSFET Differential Amplifiers

Similar DA circuits are available utilizing FET and the same procedure utilized for BJT DA analysis is applied here as well. To illustrate this, a differential mode NMOS DA circuit is given here as an example, Fig. 5.17. Its equivalent circuit is given in Fig. 5.18.

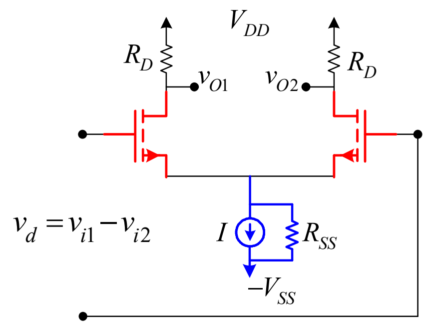


Figure 5.17: Differential mode FET DA circuit.

Carrying out similar analysis to that of BJT, we get:

$$v_{gs1} = -v_{gs2} = v_{sg2} = v_{gs}$$
$$v_{id} = v_{gs1} + v_{sg2} = 2v_{gs}$$

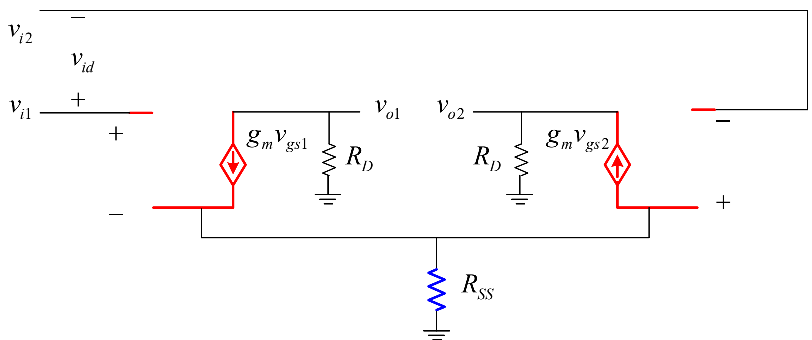


Figure 5.18: Differential mode FET DA equivalent circuit.

$$\begin{aligned} v_{o1} &= -g_m v_{gs1} R_D = -g_m v_{gs} R_D \\ v_{o2} &= +g_m v_{gs2} R_D = +g_m v_{gs} R_D \\ A_{dm} &= \frac{v_{od}}{v_{id}} = \frac{v_{o1} - v_{o2}}{v_{i1} - v_{i2}} = \frac{-g_m v_{gs} R_D - (+g_m v_{gs} R_D)}{2v_{gs}} = -\frac{2g_m v_{gs} R_D}{2v_{gs}} \\ A_{dm} &= -g_m R_D \end{aligned} \tag{5.27}$$

The common mode MOSFET differential amplifier is given in Fig. 5.19, while its equivalent circuit is given in Fig. 5.20. Analysis follows the same procedure conducted for BJT.

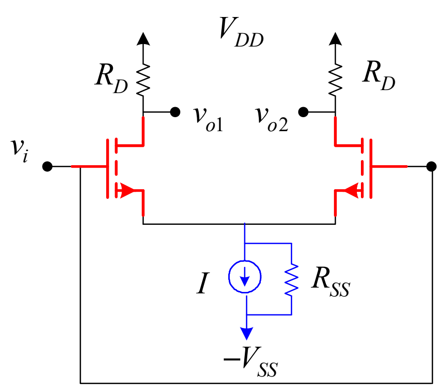


Figure 5.19: Common mode FET DA circuit.

$$\begin{aligned} v_i &= v_{gs} + 2g_m v_{gs} R_{SS} = v_{gs}(1 + 2g_m R_{SS}) \\ v_{o1} &= v_{o2} = -g_m v_{gs} R_D \\ A_{cm} &= \frac{v_o}{v_i} = \frac{-g_m v_{gs} R_D}{v_{gs}(1 + 2g_m R_{SS})} = -\frac{g_m R_D}{1 + 2g_m R_{SS}} \end{aligned} \tag{5.28}$$

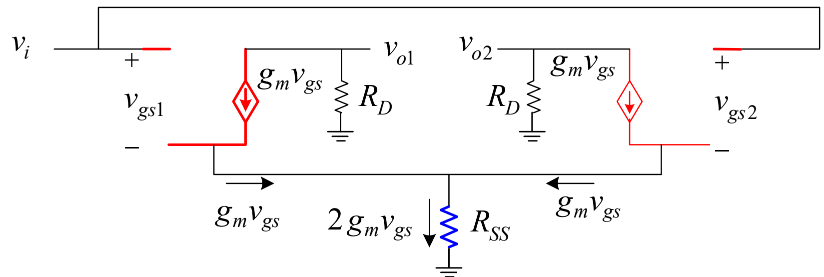


Figure 5.20: Common mode FET DA equivalent circuit.

5.3.6 Input and output resistances

Since the input is connected directly to the gate terminal of the MOSFET, then the input resistance, for both the differential and common mode connections, seen by the source is infinite i.e.:

$$R_i = R_{id} = R_{ic}\infty \tag{5.29}$$

As in the case of BJT, the output resistance for the differential mode and common mode amplifiers are simply:

$$R_{od} = 2R_D \tag{5.30}$$
$$R_{oc} = R_D \tag{5.31}$$

Example 5.6 NMOS differential amplifier driven by NMOS current source.

A differential amplifier driven by a current source is shown Fig. 5.21. Calculate:

(a) Differential mode gain

(b) Common mode gain

(c) CMMR

Solution

To calculate the differential and common mode gains (and hence CMMR), we need to calculate g_m and r_o form dc conditions.

Applying KVL and Ohm's law:

$$I_{REF} = \frac{(V_{DD} - V_{SS}) - V_{GS4}}{R_{REF}} = \frac{[5 - (-5)] - V_{GS4}}{20} = \frac{10 - V_{GS4}}{20} \tag{a}$$

Applying NMOS basic equation:

$$I_{REF} = I_Q = I_{D3} = I_{D4} = k_4 (V_{GS4} - V_T)^2 = 0.4 (V_{GS4} - 2)^2 \tag{b}$$

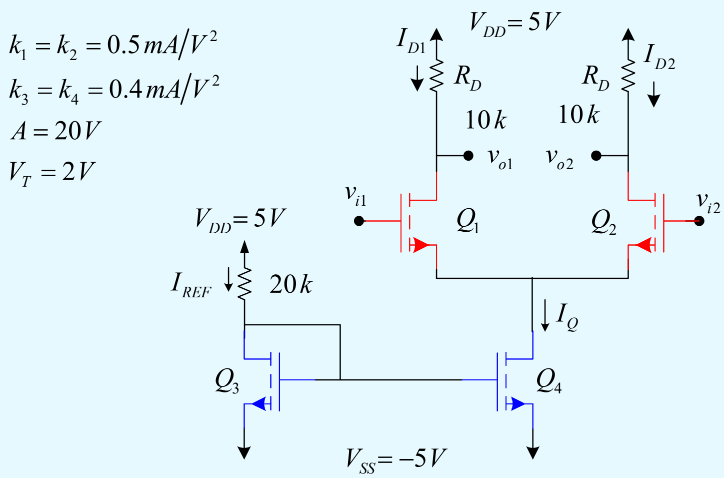


Figure 5.21: NMOS differential amplifier driven by a current source.

Equating (a) and (b), we get:

$$0.4\,(V_{GS4} - 2)^2 = \frac{10 - V_{GS4}}{20}$$
$$V_{GS4}^2 - 3.875 V_{GS4} + 2.75 = 0$$

Factoring, we get ($V_{GS4} = 0.94$ V) or ($V_{GS4} = 2.94V$). Since ($V_{GS4} = 0.94V$) is less than ($V_T = 2V$) we select ($V_{GS4} = 2.94V$). Therefore:

$$I_{REF} = I_Q = I_{D3} = I_{D4} = \frac{10 - V_{GS4}}{20} = \frac{10 - 2.94}{20} = 0.353 \text{ mA}$$

The dc currents in Q_1 and Q_2 are:

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = \frac{0.353}{2} = 0.177 \text{ mA} \quad \text{(c)}$$

Substituting in NMOS basic equation:

$$I_{D1} = I_{D2} = k_1\,(V_{GS1} - V_T)^2 = 0.5\,(V_{GS1} - 2)^2 \quad \text{(d)}$$

Equating Eg. (c) and Eq. (d) we get:

$$I_{D1} = 0.177 = 0.5\,(V_{GS1} - 2)^2$$
$$V_{GS1}^2 - 4V_{GS1} + 3.62 = 0$$

Solving this equation, we get two values for $V_{GS1} = 2.59V$ and $V_{GS1} = 1.41$ V. Since $V_T = 2V, V_{GS1} = 2.6V$ is selected for the transistor to operate in saturation. Applying basic definition of FET's g_m, Eq. 2.25 or alternative one, Eq. 2.27, we get:

$$g_m = 2k\,(V_{GS} - V_T) = 2 \times 0.5 \times (2.59 - 2) = 0.594 \text{ mS}$$

$$g_{m1} = g_{m2} = 2\sqrt{k_1 \times I_{D1}} = 2\sqrt{0.5 \times 0.177} = 0.594 \text{ mS}$$

Note that negative root is rejected as g_m cannot have a negative value.

Applying NMOS property, Eq. 2.71:

$$R_{SS} = r_{o4} = \frac{V_{A4}}{I_{D4}} = \frac{20}{0.353} = 56.65 \text{ k}\Omega$$

Therefore:

(a) Differential mode gain

$$A_{dm} = -g_{m1}R_D = -0.594 \times 10 = 5.94$$

(b) Common mode gain

$$A_{cm} = -\frac{g_{m1}R_D}{1 + 2g_{m1}R_{SS}} = -\frac{0.594 \times 10}{1 + 2 \times 0.594 \times 56.65} = -0.087$$

(c) CMMR

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{5.94}{0.087} = 68.3$$

Note that this is much smaller than that of the BJT case!!

5.4 Differential to single Ended output conversion

DA is used as first stage(s) of integrated circuits (IC's) such as op amps, to eliminate noise (since random noise adds equally to both input terminals, therefore, taking the difference output signal nulls the noise). Normally the IC is required to drive a load that is usually single ended. Differential (double ended) gain is twice that of the single-ended gain, and hence to utilize this gain the output must be converted to single ended output. A possible technique is to use a PMOS current mirror, Fig. 5.22, where a differential NMOS pair (Q_1, Q_2) is loaded with a PMOS mirror pair (Q_3, Q_4). The circuit utilizes ideal current source to simplify analysis. Circuit analysis is carried out utilizing Fig. 5.23(a) and Fig. 5.23(b).

Fig. 5.23(a) shows dc sources only and hence the two signals' inputs are short circuited. The current source I is divided between Q_1 and Q_2, i.e. each takes a current of $I/2$ flowing downwards in the direction of current I. Q_1 drives Q_3 of the current mirror with a current $I/2$. Similarly, Q_3 create a replica of $I/2$ in Q_4. The current of Q_4 is in the same direction of that of Q_2, which means that the dc current flowing in the output is zero.

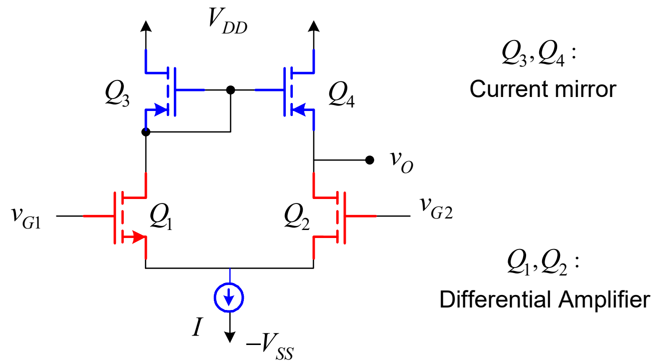
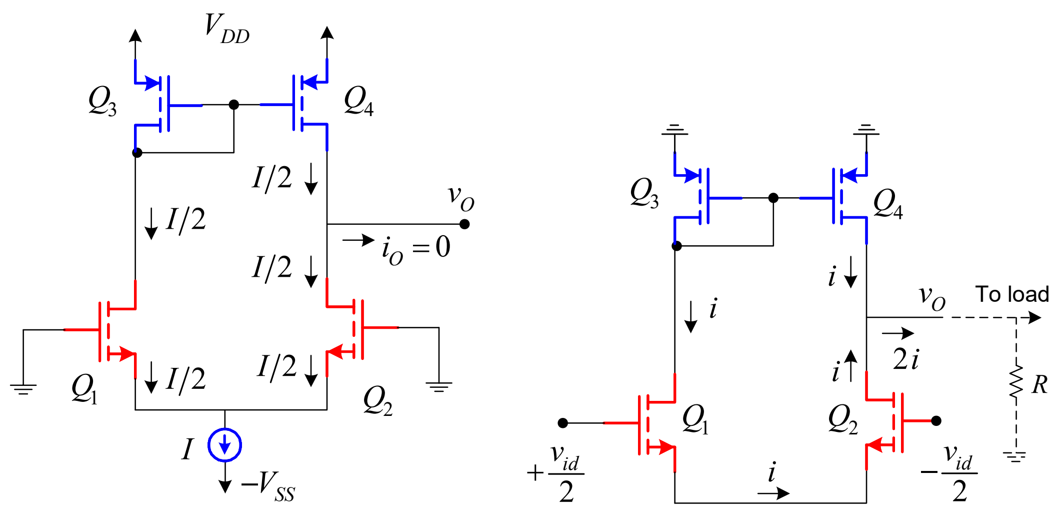


Figure 5.22: DA loaded with PMOS current mirror.



(a) DC Analysis (b) AC Analysis

Figure 5.23: Circuit analysis of DA loaded with PMOS current mirror.

AC analysis is illustrated in Fig. 5.23(b). The differential voltage is divided between the two inputs: ($v_{id}/2$) and ($-v_{id}$ /2). Q_1 drain current is flowing downwards. Drain current of Q_3 follows that of Q_1 Drain current of Q_2, which is part of the differential amplifier, flows upwards as illustrated earlier. Drain current of Q_4, which is part of the current mirror, follows the direction of current at Q_3, i.e. downwards. At the output node, the two currents from Q_2 and Q_4 add up giving an output current (2i) delivered to the load. If the load is R, then:

$$
\begin{aligned}
i &= g_m v_{gs} = \frac{1}{2} g_m v_{id} \\
v_o &= (2i)R = 2\left(\frac{1}{2} g_m v_{id}\right) R = g_m v_{id} R \\
A_v &= \frac{v_o}{v_i} = g_m R
\end{aligned}
\tag{5.32}
$$

Therefore, the differential gain has been doubled and completely transferred to a single ended output gain.

A BJT circuit counterpart is shown in Fig. 5.24. Similar analysis can be performed the BJT case, where we get:

$$A_v = g_m R \tag{5.33}$$

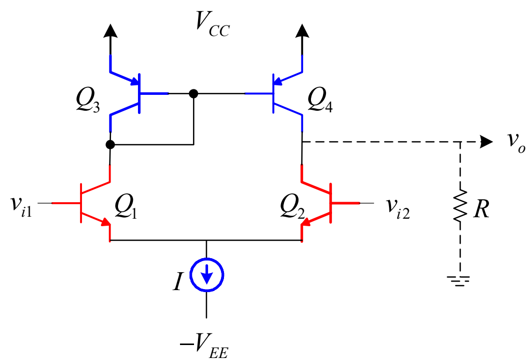


Figure 5.24: BJT DA loaded with PNP current mirror.

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